HARVARD FOREST
BULLETIN NO. 4
Richard T. Fisher, *Director*

RED OAK AND WHITE ASH

A STUDY OF GROWTH AND YIELD

BY
REUBEN T. PATTON

HARVARD FOREST, PETERSHAM, MASS.
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SUCCESSION OF TYPES

THESE studies were carried out on the Harvard Forest and surrounding woodlands. The country here is physiographically simple and consists of wide open valleys running north and south. The bed rock is gneiss and this gives rise generally to a sandy or light soil, but heavier soils also occur. The forest is not original, but is practically all second growth.

Just prior to the Civil War, the farmers of central New England began to abandon their farms, partly on account of the opening up of the West, partly because of the growth of manufactures and later because of the Civil War. The ground was very stony and rough and the climate very bitter. No extensive cultivation was possible owing to the results of the glacial period. Boulders abound, particularly at the lower elevations, which normally are the most fertile. The higher elevations are the freest from stones, but they are also the most exposed. This had been a region of dense forests when first settled and the original homes bear witness of the magnificent trees which grew in those forests. As in every case of pioneer settlement, the rich stores of timber were swept away and this rock strewn, boulder-covered land was converted to farmland. This region is peculiar in that while the soil may be regarded as agricultural, the land as a whole, owing to the boulders, must be regarded as forest land.

Once abandoned, the land was at once attacked by the forces of nature. The lightest seeded trees, white pine and gray birch, seeded up the greatest part. It would appear from many fields at present that the longer an area is left unseeded the more difficult it becomes to establish tree growth there. The soil appears to become more compacted and resistant to seed. Even gray birch, perhaps the most prolific seeder, makes slow headway on these old treeless fields. While white pine has been the most fruitful source of re-afforestation, a few fields were given back to oak, ash, and other hardwoods. Although white pine has reclothed these abandoned lands with dense stands yet to-day beneath these pines there is an abundant advance growth of all kinds of hardwoods, but particularly red oak, white ash, and red maple. These, in the seedling stage, are quite tolerant of shade and are growing freely, although the growth is much slower than in the open. The striking feature is the absence of pine in this advance growth.

We have here a replacement going on, one step towards a reversion to the original highly mixed forest. This replacement or succession is due to four factors. In the first place, the site is as good a hardwood site as it is a white pine site. The original forest was mixed hardwood with scattering pine. Secondly, the pine has formed a suitable seed bed for hardwoods. It is probable that the absence of a seed bed, equally as much as the weight of the seed, prevented the hardwoods, such as oak, chestnut, and beech, from occupying the vacant lands. An examination of germinating acorns, scattered around under a parent tree, will show that those that have lain on the surface of the ground have, in the majority of cases, damaged or decayed radicles. Under the pines the seed, aided by the agency of small animals, becomes imbedded in the litter of the forest floor and is here protected. Generally speaking, the smaller a seed, the better is its chance of forming a tree. Size of seed is as important as weight.

How successfully the pine prepared the way for the hardwoods is shown by the following example. Along an old stone fence which runs north and south there stands a row of immense old trees whose diameters are given in the following table:

TABLE I

Species	D.B.H. Inches	
Red Oak (2)	30.4	43.5
Chestnut (2)	26.0	29.8
Ash	26.4	
Basswood	26.4	
Sugar Maple	23.1	
Yellow Birch	15.7	

It is most probable that these trees were mature and bearing seed when the surrounding fields were abandoned. To the west of the fence and the row of old trees was formerly an abandoned field which grew up completely to pine. This stand was cut off a few years ago and to-day the field is completely covered with hardwoods. There are no pines in the young stand, although there are mature pines in the surrounding areas. As the prevailing winds are northwest and southwest, the seeding up of the field by the row of old hardwoods was distinctly at a disadvantage both formerly and at the present. The following table gives some idea of the composition of the young stand which is thirteen years old. The plot used for the estimation of the composition was one-twelfth of an acre and the number of trees per acre averaged 3800.

TABLE II

Species	Percent of Mixture
Ash	57
Sugar Maple	20
Red Oak	8
Gray Birch	4
Red Maple	3
Black Birch	2
Yellow Birch	2
Pin Cherry	2
Basswood	1
Miscellaneous	1

The third factor influencing this succession is the intolerance of shade shown by the pine. It has been clearly shown on the Harvard Forest that by suitable fellings a reproduction of pine up to 40,000 trees per acre can be secured. But light is necessary. In the natural stand itself, whatever pine seedlings there may be, are very stunted. On the other hand, there is usually an abundant advance growth of hardwood, as the following table will show. The plot used for the enumeration of the advance growth was .2 acre. The mature stand was pure pine, seventy years old, and averaged 300 trees per acre.

TABLE III

Species	Percent of Mixture
Red Maple	36
Red Oak	12
White Oak	12
Black Birch	12
Chestnut	10
Black Cherry	9
White Ash	6
Gray Birch	1
Poplar	1
Miscellaneous	1

The number of stems per acre of this advance growth averaged 2,600 and only stems over a foot in height were taken. In this particular area red maple is seeding in very freely. Plots ten feet square were used to count the number of red maples from one foot in height downwards. The greatest number recorded for one of these plots was 130, which is equivalent to 56,000 per acre.

Gray birch, which so freely seeds up vacant areas, does not occur at all freely in this advance growth under pine. This is due to its demands for light. In this respect it stands away from the other hardwoods and closely approaches pine. Gray birch is practically a useless tree but a vigorous grower. Where the birch has occurred in quantity in cut-over land, this has been due to lack of density of the advance growth. The birch has seeded in after the cutting of the pine, and has gained a foothold in the vacant spaces. Adequately stocked land will not be attacked by gray birch. It may be here remarked that no such thing as an average can be given for the composition of this advance growth. There are over twenty species of hardwoods occurring here. Each pine stand has its own characteristic advance growth.

The fourth factor influencing this reversion is the slow growth of pine in its early years. When the pine stand is logged all the hardwood advance growth is cut down. In the following spring the seedling and sapling stumps give forth vigorous shoots so that any pine seed which may have fallen the previous autumn germinates under a more or less dense shade. Red oak appears to have the least vigor in growth for the first year after the pine has been removed. The measurement of 330 oaks at the end of the first growing season after removing the pines gave the following results:

TABLE IV

Height — Inches	Percentage
0–10	54.9
10–20	31.5
20–30	11.2
30–40	2.1
40–50	0.0
50–60	0.3
	100%

Ash has a very vigorous growth in its early years, and in Table V is given a comparison in height growth of ash, oak, and pine for the first ten years. The areas from which the measurements were taken were close to one another and were originally covered with almost pure pine.

TABLE V

Age in Years	Height in Feet		
	Ash	Oak	Pine
1	1.5	.7	.1
2	3.0	1.6	.2
3	4.9	2.7	.3
4	6.5	4.0	.5
5	8.1	5.4	1.0
6	9.9	6.7	2.0
7	11.7	8.1	3.2
8	13.7	10.4	4.6
9	15.6	13.3	6.2
10	17.9	16.1	7.8

Although the pine type is reverting to hardwood, the incoming stands are not in themselves final. It would appear that mature hardwood stands do not provide such a good seed bed for their own seed as do the pine stands, and hence the advance growth under hardwoods is not so dense. This probably makes possible the reintroduction of the pine. The climax forest of the region is one of mixed pine and hardwoods with an understory of many species for the better sites.

The question of the yield of unmanaged second growth hardwood stands has been fully considered and the results are given in Harvard Forest Bulletin No. 2, "Growth Study and Normal Yield Tables for Second Growth Hardwood Stands in Central New England," by J. Nelson Spaeth. Since in these stands, arising from the cutting of both pine and hardwood, the most valuable trees are oak and ash and, also, since a great percentage of the young growth consists of these two trees, it is obvious that the forest of the next rotation will be largely an oak-ash stand. This type is now fully recognized by the silvicultural policy on the Harvard Forest, partly because of the great value of these two species and partly because of the prohibitive cost of reconstituting pine stands on the better soils. As these forests are situated in a region of wood-using industries where practically every load of lumber can be sold, as well as cordwood, they can be placed under intensive management. This has already been begun on the Harvard Forest, and the purpose of the present study was to obtain data for the purposes of future management. The specific questions for determination were, therefore, rotation and density as related to growth, tree form, and value of yield.

TREE FORM IN THE FINAL STAND

A. Diameter Growth

In studying the growth and development of oak and ash stump analyses of oak and ash were first made. These stumps were of trees which had produced the highest grade lumber in the district. They were of an age mainly between sixty and seventy years, with a few running up to seventy-five years. All the subsequent data were collected on sites similar to those on which the stumps occurred. The ages of the stands were similar to those of the stumps. No data were obtained for ages above seventy-five years. This was because the stands become merchantable at about seventy years and are felled at or about that age. So far, then, as local practice goes, seventy years may be regarded as the usual rotation.

Stump taper curves were made from neighboring trees and from these curves the diameter at breast high of the various stumps determined. The amount of growth in diameter for each diameter class was also ascertained.

Figure 1 gives the results of the examination of forty-two oak stumps and twenty-three ash stumps. The curves are for wood only and show some very characteristic features. In the early years, the growth of ash is much faster than that of oak. These forests have arisen mainly as sprouts from small advance growth, and hence the growth is much greater than would be the case if the tree originated from seed. Ash grows better under shade than does oak, and hence forms a better root system for the subsequent sprouts. The size of the sprouts is directly governed by the root system of the advance growth and the size of the latter is largely a question of age.

The main feature of the graphs is, however, the course of the curves at the seventieth year. Oak is still growing vigorously and is producing wood at the rate of .2″ a year, or for ten years two inches of diameter. Ash, on the other hand, is only producing wood at the rate of one inch for every ten years. Many ash trees have twenty-five rings to the last inch. This is very characteristic of ash and is due to its capacity to endure congestion or crowding of the crown. Oak, on the other hand, demands space. For these characteristics we require new terms. We might say that oak is space demanding, while ash is crowd enduring.

Since, however, increase of diameter is closely associated with clear length of bole and crown development, these two latter factors must be studied before it can be ascertained how far this rapid increase of diameter of oak can be maintained and how far the slow growth of ash may be avoided. The curves for diameter growth are in no way final in themselves since they are in no way correlated with clear bole or radius of crown. All that they do show is the characteristic type of diameter growth of each. Oak shows a desirable diameter growth while ash shows in its latter years very undesirable growth.

B. Interrelation of Diameter, Bole, and Crown

An examination was next made of the dominant trees in the age class sixty to seventy years, and for each tree the height, length of clear bole, diameter breast high and radius of crown were taken. In measuring the crown four radii were taken and the mean calculated. The measurements were repeated later on a number of the same trees, other radii being used, and it was found that the differences resulting were small and could be neglected. Trees having very irregular crowns were rejected.

The term clear bole was taken to mean the length to any branch, dead or alive, beyond which there was not a clear log length. This gives correct results so far as the uncultivated forest is concerned, but it does not correctly indicate the length of clear bole obtainable under systematic control. However, it has been retained as introducing a conservative element into the conclusions. This will be referred to again.

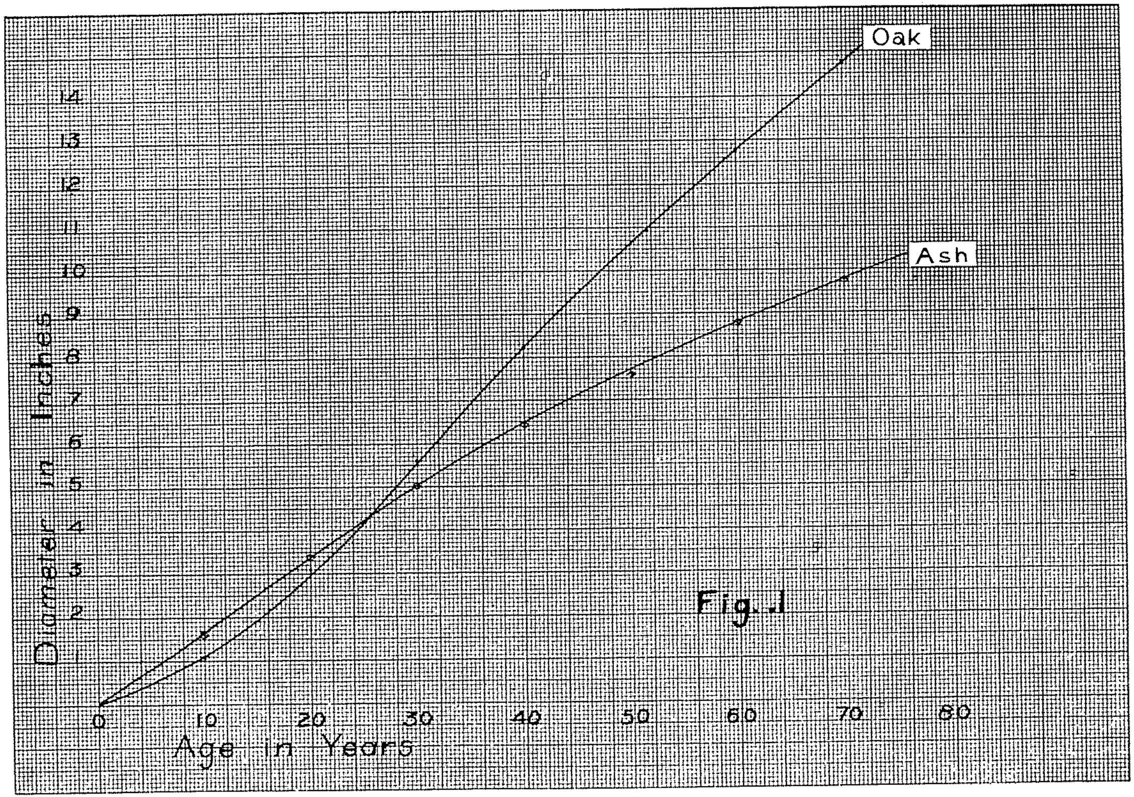

Oak
Ash
Diameter in Inches
1
2
3
4
5
6
7
8
9
10
11
12
13
14
0
10
20
30
40
50
60
70
80
Age in Years

Fig. 1

In the following table are given the results for ash. The trees are grouped by one-inch diameter classes. The figures given are the actual averages obtained in the field.

TABLE VI. ASH

D.B.H. Inches	Height in Feet	Bole in Feet	Crown Radius in Feet	Equivalent Trees per Acre
7.0	72	47	5.5	459
8.1	74	46	6.0	385
9.0	79	47	6.6	318
10.1	79	45	7.1	275
10.9	82	48	7.7	234
12.0	80	44	8.7	183
13.1	80	44	10.0	139
Basis	109 trees.	Age 60—70 years.		

It will be seen that though the spacing of the trees as given by the crowns amounts to a difference of nearly one hundred per cent yet the total height averages about the same for all classes, except the first two. This implies that up to a certain point, at all events, closeness or openness of the stand does not affect height growth.

The average length of clear bole in all is comparable. This length averages around forty-five feet but in a managed forest it could be taken at fifty feet, or even at fifty-five feet. Ash trees felled on Harvard Forest, from 8″ to 12″ in diameter have given a clear length of sixty feet. That the average length of clear bole is practically the same for all the crown radii given is important since it indicates that the limit of thinning an adequately stocked young forest of ash has not yet been reached; or, in other words, a system of thinning which would produce these results is not severe enough to retard natural pruning. Each row of figures in the preceding table represents a possible end point at seventy years for any system of progressive thinning. Which is the best, however, must be settled by the objective of the forester. In general, this objective is the greatest money return. The financial yield is itself governed by the total yield and value of that yield.

In Table VII is given the board foot content per acre for an ideal stand of each diameter class. The bole has been taken at forty-five feet and all the trees have been assumed to have the same taper.

TABLE VII. ASH

D.B.H.	Yield per Acre Board Feet
8.0	29,300
9.0	27,400
10.0	25,400
11.0	24,700
12.0	24,400
13.0	23,200

The yield decreases with increasing diameter of stem. In the table we have not considered cordwood, which would increase the yields of the larger diameter classes. This, however, need not be considered as the volume of the wood involved is slight and the aim of the investigation was to ascertain the greatest financial return from saw timber.

Before considering the ash further, the results for oak will be given. It has already been mentioned that ash differs materially from oak when grown in high forest. This is fully borne out by a consideration of Table VIII, which gives for oak the relations between diameter, height, clear bole, and crown radius.

TABLE VIII. OAK

D.B.H.	Height in Feet	Bole in Feet	Crown Radius in Feet	Equivalent Trees per Acre
11.2	78	40	8.8	179
12.1	80	38	9.7	147
13.0	82	37	10.6	123
14.0	81	34	11.4	107
15.0	83	33	12.5	89
16.0	84	31	13.6	75
17.1	83	29	14.8	63
17.9	84	28	15.5	58
19.0	83	27	16.6	50
20.0	83	22	17.5	45
Basis	193 trees.		Age 60–70 years.	

With two exceptions, the figures are the actual averages obtained for each diameter class. This table shows that there is a much greater variation in the case of oak than is the case with ash. Yet both species occur together in the same stand. Oak demands space and struggles to obtain it. Its lower branches persist much longer than do those of ash, and this despite the fact that ash appears to be much more tolerant of shade in the seedling stage. Oak will maintain a lower branch and thrust it out towards a source of light where ash would sacrifice the branch.

Oak is similar to ash in that the height growth is not affected by the density of the stand. This is true, of course, only within certain limits; but so far as the tables given go those limits have not yet been reached.

While total height is not affected by the degree of spacing, the length of clear bole is seriously decreased by the wider spacing. It will be seen that there is a gradual decrease in length of clear bole with increase of crown radius. When the crown canopy closes the lower branches are killed, and it comes about where the density was small in the early years that there is a big difference between length of clear bole and length to first living branch. In the larger diameter classes length of clear bole and length to first living branch usually coincide. This will be dealt with later when the trees will be grouped under bole classes of different lengths.

In Table IX is given the yield per acre for an ideal stand of each diameter class given in the last table. The yields are taken from mill tally studies made on the Harvard Forest. The clear bole has been divided into the standard log lengths of the district eight, ten, and twelve feet and the board feet contents have been read from the log tables.

TABLE IX. OAK

D.B.H. Inches	Board Feet Per Acre
11	23,000
12	19,100
13	17,700
14	16,400
15	15,700
16	14,900
17	13,400
18	13,100
19	11,650
20	10,750

As in the case of ash, the yield steadily decreases with increasing diameter of stem. But yield is not the only factor, for the lumber from the different diameter classes has different values and before discussing the value of the yield, the variation in price according to the average width of board will be considered.

C. Relation of Width to Price

It is well enough known that the narrower the board used in any particular industry, the greater the percentage of waste. Hence, it is to be expected that the value will bear some relation to the percentage of useable lumber in a board. The percentage of waste rapidly decreases as the boards increase in width until, above a certain width, the difference in the amount of loss is negligible. Hence, with wide boards the value ought not to change.

In local hardwood using industries, this was found to be the case and prices were obtained for oak from both chair manufacturers and wholesale dealers. It was found impossible to get a series of values for ash, partly because it is not greatly used locally and partly because it is used in certain specific industries where only particular qualities or sizes are required, such as tennis rackets, baseball bats, oars, motor cars, airplanes, etc. On account of its use for these special purposes higher prices are paid for ash than for oak. Oak, on the other hand, is used locally in large quantities and in all sizes. In the chair industry where most of the red oak is consumed the logs are sawn into plank 2″ to 4″ thick, round edge. For first quality, the timber must be practically clear, free from defects and straight in grain. Width of ring, or, in other words, whether the timber be fast or slow grown, is of no importance and does not affect price. In an examination of lumber piles, the widest rings seen in oak were half an inch. The source of this lumber was not known, except that it was not obtained in New England. In the various operations in the factories, such as turning, bending or polishing, width of ring was of no importance. This is not the case with ash, for width of ring is an important factor in certain industries. These, however, were not investigated. The highest price paid for oak is for plank averaging about 10″ and upwards in width. Wide planks are not favored owing to the greater difficulty of handling them. The smallest average width for which a price was quoted was six inches, and the price paid for this ranges from about fifty-three per cent to sixty-three per cent of the price of the 10″ lumber. There is an abrupt rise to the price of 7″, and each succeeding rise is less as the limiting value is reached. If we consider the highest price paid as one hundred per cent, the prices of the various sizes would be as in the following table:

Average Width	Percentage of Highest Price	Approximate Fraction Value
10	100	1
9	97–95	1
8	92–89	9/10
7	82–77	8/10
6.5	68–60	7/10–6/10
6.0	62–53	6/10–5/10
5.5	34–18	3/10–2/10
5.0	22–5	2/10–1/20

The percentage values of 5 and 5.5 inch lumber have been calculated from the prices obtained. It would appear from this that timber averaging five inches in width, if used in the same industry as the wider material, has very little value.

Timber averaging four inches would have no value in the same industry.

It will be observed that prices rise for each inch class by gradually decreasing increments and proceed to a limiting value. The increases in value when the boards become wide are very small, and are neglected commercially. The law underlying the increases in value appears to be that the increment in value from one inch class to another is approximately a definite fraction of the difference between the value of the lower class and the maximum or limiting value. This fraction appears to be about one-half. For example, if the maximum value of oak is $60 a thousand, and the price of six-inch be $40 then the value of the seven-inch would be $50 a thousand.

D. Average Widths and Values of Yields

Since increase of the average width of the lumber is the important factor in value, it is of interest to know what average width of $2\frac{1}{8}''$ plank each diameter class of tree will produce. In Table XI is given the average width of ash lumber sawn $2\frac{1}{8}''$ with $1\frac{1}{8}''$ sidings.

Locally, all good grade lumber is cut into planks with a thickness of $2''$ and over. The $2\frac{1}{8}''$ is surveyed as $2''$ and the $1\frac{1}{8}''$ sidings as $1''$. For thickness $2''$ and over the price does not vary.

TABLE XI. ASH

D.B.H. Inches	Av. Width of $2\frac{1}{8}''$	Board Feet Per Acre
8	4.9	28,600
9	5.5	26,800
10	6.1	23,600
11	6.7	24,000
12	7.3	22,400
13	7.9	22,100

The board feet per acre include the sidings and the percentage of sidings increases with decrease in the diameter of the log. As it was found impossible to obtain a series of prices for ash for the various average widths of the lumber, the fractional values given for oak in Table X have been used. These have been multiplied by the board feet per acre to obtain what has been called the relative value per acre. The yield per acre is considered as consisting wholly of 2″ plank.

The results are given in Table XII.

TABLE XII. ASH

D.B.H. Inches	Fractional Value	Board Feet Per Acre	Relative Value Per Acre
8	1/5	28,600	5,720
9	3/10	26,800	8,040
10	3/5	23,600	14,160
11	3/5	24,000	14,400
12	4/5	22,400	17,920
13	9/10	22,100	19,890

The largest diameter class has the greatest relative value, and the relative value increases from the smaller diameter classes upwards. From Table XII it is apparent that the most profitable diameter class to produce is the largest. The number of trees having a diameter of 13″ at seventy years was very small and, therefore, it is at present doubtful if that diameter could be generally produced at that age. The 12″ class is frequent and, therefore, we may at present consider this to be the ideal diameter to be produced in seventy years.

In Table XIII is given the average width of lumber from the oak according to lengths of bole and diameters given in Table VIII.

TABLE XIII. OAK

D.B.H.	Av. Width 2⅛″ Plank	Yield Per Acre Board Feet
11	6.4	23,000
12	7.3	19,100
13	8.1	17,700
14	8.8	16,400
15	9.5	15,700
16	10.2	14,900
17	10.9	13,400
18	11.5	13,100
19	12.1	11,650
20	12.7	10,700

Since the maximum price is paid for ten-inch lumber and since all stands of diameter classes from sixteen inches onwards have an average width of 10″ and over and produce progressively less lumber per acre, there is no need to consider these further. The larger diameter trees could not be considered in any controlled forest, since, in order to obtain a large diameter, length of bole would have to be sacrificed. In these larger trees, as has already been mentioned, the top of the bole ends at the first living branch. Therefore, the large diameter could not be secured with a longer bole in a rotation of seventy years. There is another consideration, and this is that these larger trees are seventy years of age or slightly over, while the smaller diameter classes are about sixty-five years. However, for the present all are being considered as of the same age.

In Table XIV is given the total value per acre of a stand of the various diameter classes. The amount of 1⅛″ sidings is taken from local mill practice. The prices given for the average width have been averaged from actual prices obtained. The limits are actual ruling prices.

TABLE XIV. OAK

D.B.H. Inches	Av. Width 2⅛″ Plank	Board Ft. Per Acre	Yield 2⅛″	Price M. Ft. $	Value $	Yield 1⅛″	Price M. Ft. $	Value $	Total Value $
11	6.4	23 000	19,550	36	704	3450	25	86	790
12	7.3	19,100	16,235	46	747	2865	"	72	819
13	8.1	17,700	15,050	51	768	2650	"	66	834
14	8.8	16,400	14,760	53.5	790	1640	30	49	839
15	9.5	15,700	14,130	55	777	1570	"	47	824
16	10.2	14,900	13,410	55	738	1490	"	45	783

The average width of lumber from the 15″ class was 9.54 and, therefore, the price given to this class was the maximum. The highest return comes from the 14″ class, followed closely by the 13″; but there is, on the whole, not much difference between the total values.

The length of bole of the 14″ class was thirty-four feet, but it is to be expected that under systematic management the forest will be able to produce a much longer bole than this.

E. The Final Objective

From the previous discussion of ash, it was seen that the greatest diameter class gives the greatest return. However, since the 13″ class in ash was not frequently found, it is as yet doubtful if this diameter can generally be produced in seventy years. The next class, the 12″, is abundant. The age of this class was between sixty-five and seventy years. Sample trees which were felled showed that a diameter of 11.6″ was produced in sixty-five years. At the same rate of growth occurring during the previous five years, a diameter of 12.4″ would be produced in seventy years. These sample trees had clear boles of fifty-five feet. In Table VI the 12″ diameter was associated with a crown radius of 8.7. From a graph of crown radii against diameters of stem we find that a diameter of 12.4″ would be associated with a crown radius of 9.1 feet.

We may say then that the trees of the final harvest of ash at seventy years would average 12.4″ diameter, 9.1 feet crown, fifty feet bole, and a height of eighty-two feet.

In the case of oak where the bole gradually decreases with increasing diameter, the difference between clear length of bole and length of trunk to first living branch is very great, especially in the medium diameter classes. In ash this difference is small. Hence in oak with each diameter class there is a great variation in length of bole, and this variation diminishes as diameter increases. That portion of the bole which bears dead branches could be greatly reduced if the forest were adequately stocked in its early life.

Boles fifty feet in length occur, and this is the desirable length. The largest clear bole found in oak was fifty-four feet, while the longest in ash was sixty-eight feet. It may be said in passing that if a clean bole of fifty feet can be obtained on a ten-inch tree it can be secured for the higher diameters and later years.

If the trees in Table VIII be grouped by length of clear bole we get the results given in Table XV.

TABLE XV. OAK

Bole Feet	Diameter Inches	Crown Radius Feet
50	13.4	10.4
45	13.6	11.0
40	13.8	11.3
35	13.9	12.0

It will be observed that the 11″ and 12″ diameters have become merged into the 13″ and 14″ classes. It will be noticed also that while the difference between the diameters amount to 3.7 per cent, the difference in crown radii amounts to 15.4 per cent. Within narrow limits each diameter can be produced by different sized crowns. Generally, however, the larger crown is associated, as in the table, with a shorter bole. The extra energy developed by the larger crown is used up in forming branch wood.

Sample trees with boles above forty-five feet which were felled as well as the stump analyses showed that a diameter of 13.4 to 14.9 was produced in sixty-five years. At the rate of growth occurring in the 13.4″ tree during the decade prior to felling, a diameter of 15″ would be produced in seventy years. It was found, by the method to be discussed in the next section, that a crown of 11.2 feet radius would be associated with a fifteen-inch diameter and fifty-foot bole. In the 15″ diameter class the longest clean bole was forty-nine feet and was associated with a crown radius of only nine feet. The average for the class was 12.5 feet, so that the radius 11.2 feet is well within the lower limit of the range found in nature. It may be again stated that in each diameter class the longest boles are associated with the smallest crown radii. The final harvest of oak then will consist of trees averaging, at seventy years, 15″ in diameter, fifty feet bole, 11.2 feet crown, and a height of eighty-five feet. A tree of these dimensions will produce an average width of 9.1′ for 2″ plank.

Having selected the type of tree that is to form the final harvest, the question arises as to how to treat these areas, which have reverted to hardwood, so as to produce the desired crop. This is primarily a matter of the proper control of the density throughout the life of the stand. There was an abundance of material for study below twenty years and above sixty years, but there was rather a scarcity of material between these ages. It was, therefore, necessary to follow in part some other line of study in order that the deficiency might be overcome. It was decided to make a study of crown development both from living material and from existing yield tables. If the area of the ideal crown for any age be known, then the corresponding number of trees per acre could be found, and thus a criterion established for periodic thinnings.

YIELD TABLES

A. Expansion of Crown

It has been noticed in Tables VI and VIII that stem diameter is a function of crown radius. In general, the larger the crown the larger the diameter of the stem. Growth of crown is essentially different from either height growth or diameter growth. We have already seen that within at least certain wide limits growth in height is not affected by the density of the stand. For the variations in density in ordinary forest practice, height growth may be said to be beyond the control of the forester. Diameter, however, being a function of the crown radius, may be varied by varying the crown radius, or, in other words, by varying the density of the stand. This latter is secured by various degrees of thinning.

With open grown trees height growth, diameter growth and crown growth follow much the same curves of growth. If we consider an open grown oak or pine, the longest branches are very low down and may even rest on the ground. Pine branches are easy to study and these show that the expansion of the crown is at first very slow, then very rapid for a few years, and then after this rapid growth there is a very gradual decline. This is precisely the same type of growth as for height and diameter.

In a forest, however, crown growth is entirely different from other forms of growth, and may be put into a class by itself. In the forest the crown is engaged in a struggle for existence; it is constantly surrounded by forces opposing its development, while both height and diameter are free. The crown has either to overcome or be overcome, to win or to be subdued. If we consider pine for the sake of simplicity, we find that the attack by the crown is carried on and maintained in intensity by reinforcements in the form of new branches. In both height and diameter development, the new additions are always placed upon the old, but in the crown struggle new units are constantly being added and the old ones pass to their death. In the open grown specimen the position of the crown is fixed, but in the forest tree the crown moves up the stem. Since the growth of the crown then is constantly impeded and resisted, it is to be expected that its curve of growth will differ materially from that of diameter or height. Since all forests, no matter what their composition, are engaged in the same struggle, it is not unreasonable to expect that some common law holds for the development of the crown. Naturally, development of the crown means the constant elimination of trees and the lessening of the density. Although all engage in this struggle, it does not say that all trees engage in it with the same intensity. It has already been noted that oak is space demanding, while ash is crowd enduring. The feebleness of the struggle carried on by ash is reflected in the decrease of the rate of diameter growth as already mentioned. White pine seems to stand midway between those two trees. From a study of the crowns of various species, it was assumed that the expansion of the crown was a linear function of time. To see how far this might be true an examination of a very large number of European yield tables was made and the crown radius determined for each decade. In the calculation the crown has been assumed to be circular. Figures 2, 3, 4 give the radii of the various crowns plotted against time. Figure 2 is taken from Grave's *Mensura-*

tion. Figure 3 is taken from *The Reports of the Swedish Institute of Experimental Forestry*, 1916–17. Figure 4 is from the *Allgemeine Forst und Jadg Zeitung*, vol. 84, p. 266.

Only one graph in those given is a distinct curve. Many crown graphs are distinct curves. Yet the majority show a straight line. In many the curve is so slight as to be approximately a straight line. This is the case in each of the three site qualities given by Fleury in *Ertragstafeln für die Fichte und Buche der Schweiz*, Zurich, 1907, and also in the four site qualities given by Wimmenauer for oak in *Allgemeine Forst und Jadg Zeitung*, vol. 89, p. 261. Slight curves might also be drawn in Figures 2, 3, and 4.

Since the expansion of the crown can be represented by a straight line, it follows that the crown growth is a linear function of time. Before applying this law to the data given for oak and ash it is as well to consider the equation for the the straight line. The statement that the expansion of the crown is a linear function of time refers only to the period of time covered by the average rotation. The general equation for the straight line is:

$$r = b + mt \quad \ldots\ldots\ldots\ldots \quad (1)$$

where r = radius of the crown in feet
b = intercept on the axis of r
m = tangent which the graph makes with the t axis
t = time in years

Now b may be either positive or negative. If b be positive it implies either that there was already a crown existing when the forest commenced or else that the greatest period of growth was in its earliest years. In the latter case the curve of expansion would be convex upwards until the straight line was reached. Neither assumption is true; and, therefore, b cannot be positive. Such graphs arise through the stand being either insufficiently stocked in its early life or else through its being too congested later in life. In white pine stands, both factors are operating and from most stands a

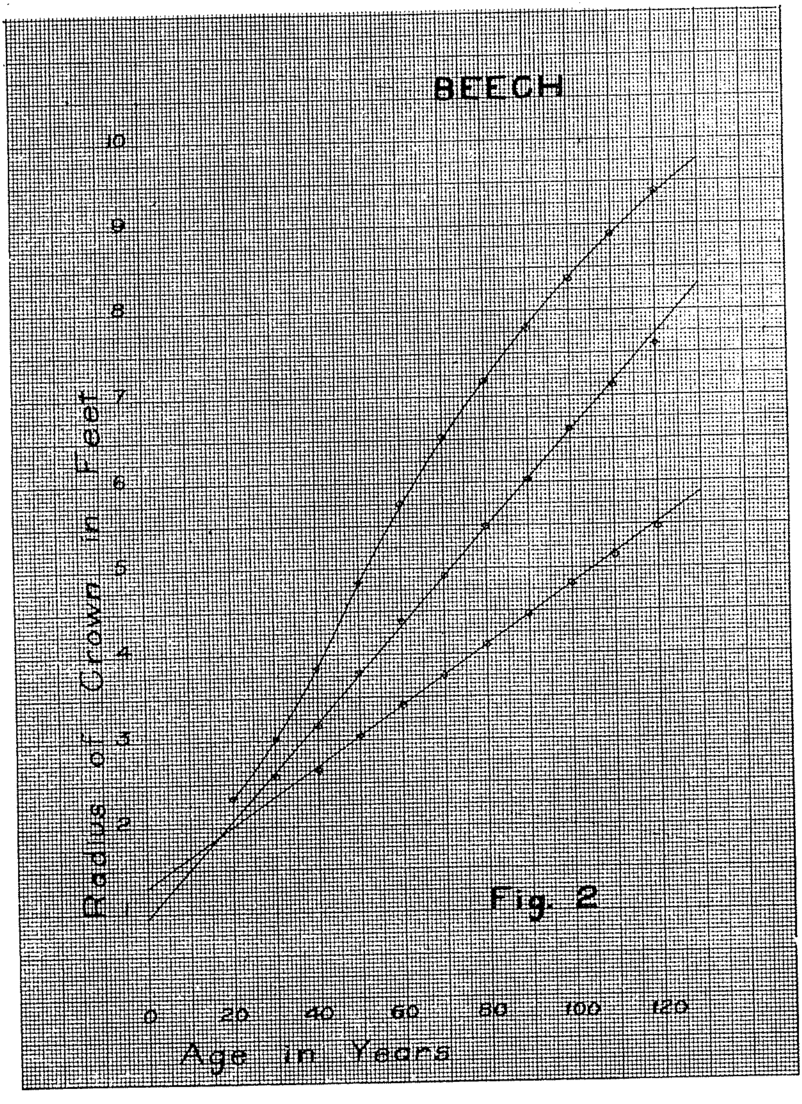

BEECH
Radius of Crown in Feet
1
2
3
4
5
6
7
8
9
10
0
20
40
60
80
100
120
Age in Years

Fig. 2

graph would be obtained cutting the vertical axis above the origin. It may be here remarked that the congestion of the crown canopy at the later periods of the forest's life may so restrict the crowns that any thinning experiments carried out on such a forest would yield negative results. As has been shown already, the smaller the crown for any given age, the smaller the diameter. The smaller the diameter for any given height the less the rigidity of the stem. The thin stemmed, small crowned trees sway in the wind and their crowns rub off all the lateral buds from their own and the neighboring crowns. This occurs particularly in ash and pine. Such trees cannot respond actively, if at all, if the stand be thinned. A tree does not differ from any other biological unit and can only respond to a stimulus when kept in a condition to do so.

For any graph b cannot be positive, but may be either zero or negative. If zero it assumes that crown expansion proceeds regularly through the life of the stand. If b is negative, it implies that growth at the early period was slow and that a concave curve joins the straight line to the origin. In all four site qualities of oak as given by Wimmenauer b is negative. It will be seen that in the case of larch, Figure 3, the m values are not widely different. In some sets of graphs the m values are all the same. This would imply that the trees on all site qualities carried on the struggle for existence with the same intensity. This cannot be considered as correct, for since a lower site quality is indicated by a smaller height, a smaller diameter and a smaller volume, it follows that trees on a lower site quality carry on the struggle for existence with less intensity than those on a higher site quality. We may conclude, therefore, that the higher the site quality the higher the value of m; and, hence, the graphs of crown expansion for the site qualities of any particular species will not be parallel.

Now r is connected in the equation

$$\pi r^2 n = A$$

When n = number of trees per acre
and A = acre in square feet

Substituting for r in (1) we have

$$\sqrt{\frac{A}{\pi n}} = b+mt \ . \ . \ .$$

which gives

$$n\,(b+mt)^2 = \frac{A}{\pi} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2)$$

If b be zero, then the equation simplifies into

$$nt^2 = \frac{A}{m^2\pi}$$

Since A and π are constants and m is also a constant for any one species on any one site we may replace the right hand member of the equation by the constant C.

Hence we have

$$nt^2 = C \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

This last equation says that the number of trees at any time multiplied by the square of the time gives a constant value. The simplified formula closely agrees with Wimmenauer's now celebrated Scotch pine experiments.

If the expansion of the crown be a linear function of time, then if any two points of the graph are known, the graph itself is known. From such a graph the number of trees per acre for any given year can readily be calculated by dividing the area of a single crown in square feet into the number of square feet in an acre. If, however, the graph passes through the origin then a good deal of calculation may be saved by merely calculating the trees per acre for any one year and then using formula (3) for the other years.

For both oak and ash two points on the graph of the crown expansion were obtained. From young stands, presumably fully stocked, the following dimensions have been taken:

TABLE XVI

Species	D.B.H. Inches	Height Feet	Crown Radius Feet
Oak	1.2	18	1.4
Ash	1.2	16	1.2

It has already been mentioned that the final objectives for oak and ash at seventy years are as follows:

TABLE XVII

Species	D.B.H. Inches	Height Feet	Bole Feet	Crown Radius Feet
Oak	15.0	85	50	11.2
Ash	12.4	82	50	9.1

From stumps whose diameters at breast high were within an inch of the expected or ideal final diameter, the curves of diameter growth for both oak and ash were obtained. From these curves the diameters and heights, falling between the limits given in the two preceding tables, were taken. From felled sample trees the curves of height growth were obtained similarly.

The tabulated results were as follows:

TABLE XVIII

Age in Years	Ash		Oak	
	D.B.H. Inches	Height Feet	D.B.H. Inches	Height Feet
10	1.2	18	1.2	16
20	2.7	36	2.7	37
30	4.8	53	4.5	57
40	7.0	68	6.8	68
50	8.8	77	9.5	75
60	10.6	81	12.2	81
70	12.4	82	15.0	85

In the case of ash, if on the graph we join the point 9.1 feet, crown radius at seventy years, with the origin, it would give a crown radius of 1.3 at ten years. This agrees closely enough with the radius in Table XVI, which was 1.2 feet. Taking the value 1.3 feet for ten years, this would give a radius of 3.9 at thirty years. Two stands of this age were examined and the results are given in Table XIX.

TABLE XIX. ASH

D.B.H. Inches	Height Feet	Bole Feet	Crown Radius Feet
4.8	55	31	4.1
4.9	52	29	3.9

Both of these agree with the diameter and height of Table XVIII for thirty years. Hence, we may assume that the calculated crown radius of 3.9 is the correct one for thirty years.

Measurements in a stand of from fifty to fifty-five years gave a crown radius 6.8. This latter would be correct for fifty-two years. The average height of seventy-three is fairly comparable with that given in Table XVIII.

In the case of oak if we join the point 11.2 feet at seventy years to the origin we get a crown radius of 1.6 feet at ten years. There is not very much difference between this and the computed 1.4 feet. However, oak stands need to be kept dense in order to prevent as much as possible the large development of side branches. The graph connecting the points 11.2 feet at seventy years and 1.4 at ten years will put the r axis below the origin.

A stand forty-three years of age with good boles and well formed crowns showed an average diameter of 7.1 inches and an average height of fifty-nine. The crown radius was 6.7 feet. While diameter and crown development agree generally with the computed measurements for oak on the better sites, the height is considerably lower and, therefore, the site may be considered to be inferior to those from which the measurements in Table VIII were obtained.

A stand whose age was from fifty to fifty-five years showed almost perfect agreement with the measurements in Table XVIII. Trees in this stand averaging 9.2 inches for the nine-inch diameter class had an average bole of forty feet, a height of seventy-six feet and a crown radius of 8.0 feet. In this same stand the ten-inch diameter class, averaging 10.1 inches, gave an average height of seventy-six feet, a bole of forty-two feet and a crown radius of 8.9 feet.

While the stands available for the ages twenty to sixty years were not by any means abundant the results obtained from such stands are in agreement with the results derived from the assumption that the crown expansion is a linear function of time.

B. Yield Tables

From the data given in the preceding pages the yield tables below have been constructed. As has already been remarked, pure stands do not occur and, therefore, a percentage composition is given in Figure 5, so that the yield in any mixed stand may be ascertained by calculation. It is also true that pure ash-oak stands do not occur, but the percentage of other trees in the stand will depend greatly on the management. Where the percentage of such trees as red maple and gray birch is large at the commencement of the life of the stand, the removal of these will form the greater part of the early periodic thinnings.

For the purposes of this yield table, basswood, beech, and chestnut may be considered as oak in regard to number per acre. Chestnut, however, may now be considered a thing of the past owing to the ravages of the disease *Endothea parasitica*. Both basswood and beech are space demanding like oak. Neither of these trees occurs at all freely in the mature forest, but it is probable that they were present in the original forest in a greater percentage than to-day. Sugar maple, black cherry, red maple, black birch, and yellow birch may be regarded as ash as regards number per acre. All of these trees are crowd enduring and all form clear boles like ash. The difference in the height growth of oak and ash at any period is small and neither tree is in any danger of being overtopped by the other. There is a rather remarkable difference in their growth at about the sixteenth year and onwards. The annual height increment of ash, which is more regular than is the case with oak, suddenly becomes slower, so that the general increment at about the seventieth year is only one or two inches, while in oak it is about four inches. This sudden decline in the rate of growth also occurs in white pine and apparently the height above ground or age at which it takes place varies according to the site.

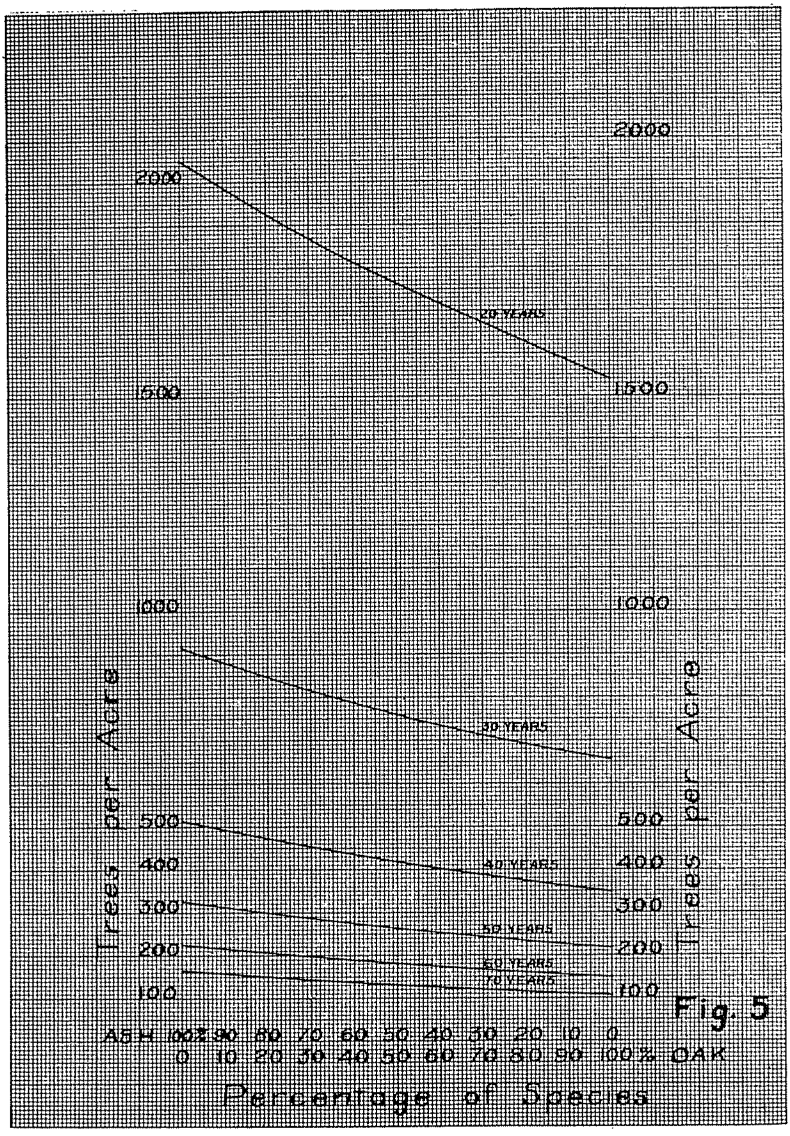
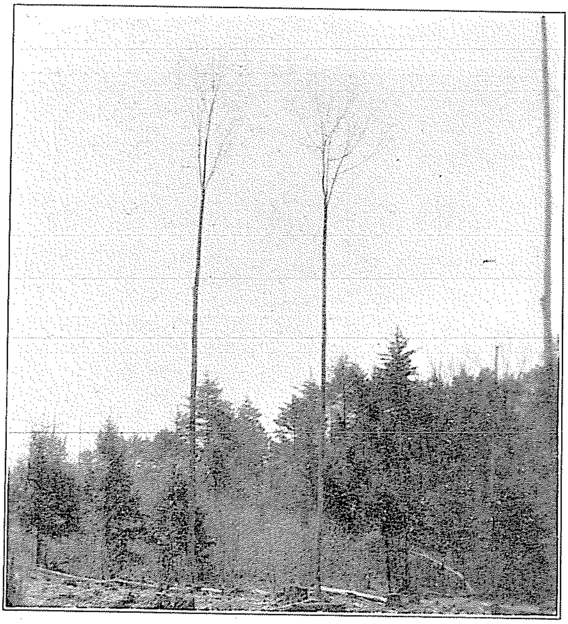
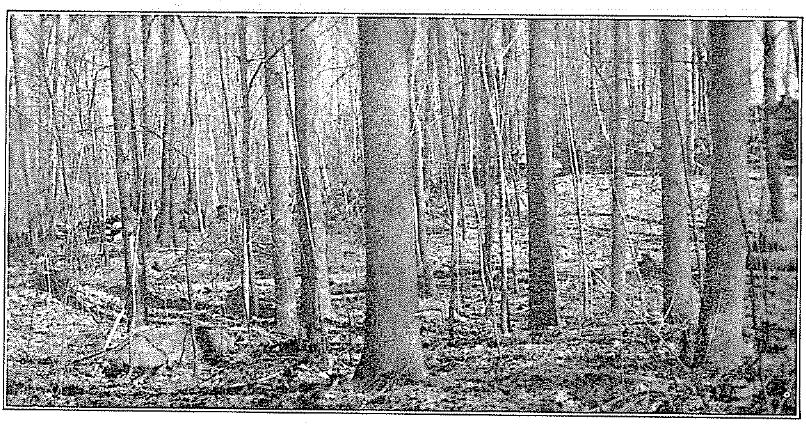

2000
2000
20 YEARS
1500
1500
1000
1000
30 YEARS
Trees per Acre
Trees per Acre
500
500
400
400
40 YEARS
300
300
50 YEARS
200
200
60 YEARS
70 YEARS
100
100
ASH 100% 90 80 70 60 50 40 30 20 10 0
0 10 20 30 40 50 60 70 80 90 100% OAK
Percentage of Species

Fig. 5



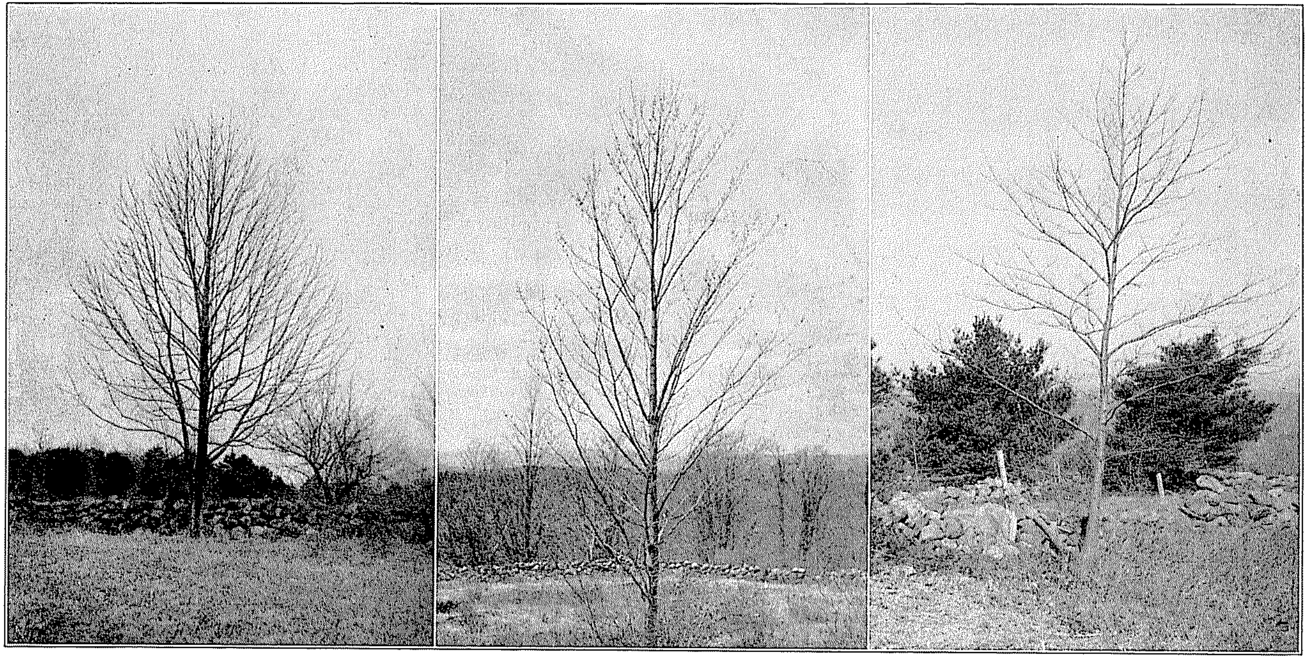
TYPICAL ASH TREES

Grown in close canopied high forest. Height of trees: left, 87 ft.; right, 82 ft.



TYPICAL ASH STAND

Age 65 years. D.B.H. central tree, 12″.



WHITE ASH RED MAPLE RED OAK

In both Maple and Ash the branches are acutely angular, while in Oak they approach the horizontal. Both the former readily lose their lower branches when the stand is dense. Both are crowd-enduring. Oak is space demanding and does not readily lose its lower branches.